CBCS SCHEME

USN | | | | | | | | | | |

18EC44

Fourth Semester B.E. Degree Examination, Dec.2023/Jan.2024
Engineering Statistics & Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. Define a random variable and briefly discuss the following terms associated with random variable.
- (i) Sample space.
- (ii) Distribute function.
- (iii) Probability mass function.
- (iv) Probability density function. **(06 Marks)**

b. The pdf for random variable Y is given by,

$$f_y(y) = \begin{cases} 1.5(1-y^2), & 0<y<1 \\ 0, & \text{otherwise} \end{cases}.$$

(i) What are the mean?
(ii) What are the mean of square?
(iii) What are the variance of the random variable Y? **(06 Marks)**

c. Define an uniform random variable. Obtain the characteristics function of an uniform random variable and using the characteristic function derive its mean and variance. **(08 Marks)**

OR

2 a. The probability density function of a random variable 'x' is defined as,

$$f_x(x) = \begin{cases} K, e^{-4x} & x>0 \\ 0 & x \le 0 \end{cases}.$$

Find
- (i) Constant K.
- (ii) $P(1<x<2)$
- (iii) $P(x \ge 3)$
- (iv) $P(x<1)$ **(08 Marks)**

b. Given the data in the following table :

k	1	2	3	4	5
Z_k	2.1	3.2	4.8	5.4	6.9
$P(Z_k)$	0.19	0.22	0.20	0.18	0.21

(i) Plot the pdf and the cdf of the discrete random variable z.
(ii) Write expressions for $f_Z(2)$ and $F_Z(2)$ using unit delta function and unit step function respectively. **(06 Marks)**

c. Define Poisson distribution. Obtain the characteristic function of a Poisson random variable and using the characteristic function derive its mean and variance. **(06 Marks)**

Module-2

3 a. The joint pdf $f_{xy}(x,y) = C$, a constant, when $(0 < x < 2)$ and $(0 < y < 3)$ and is 0 otherwise.
   (i) What is the value of constant C?
   (ii) What are the pdfs for X and Y?
   (iii) What is $F_{XY}(x, y)$ when $(0 < x < 2)$ and $(0 < y < 3)$?
   (iv) What are $F_{XY}(x, \infty)$ and $F_{XY}(\infty, y)$?
   (v) Are x and y independent? **(10 Marks)**

b. The mean and variance of random variable x are -2 and 3 ; the mean and variance of y are 3 and 5. The covariance Cov(xy) = -0.8. What are the correlation coefficient ρ_{XY} and the correlation E[XY]. **(06 Marks)**

c. Define correlation coefficient of random variable X and Y. Show that it is bounded by limit ± 1. **(04 Marks)**

OR

4 a. The zero mean bivariate random variables X_1 and X_2 have the following variances : $Var[X_1] = 2$ and $Var[X_2] = 4$. Their correlation coefficient is 0.8. Random variables Y_1 and Y_2 are obtained from,
$Y_1 = 3X_1 + 4X_2$, $Y_2 = -X_1 + 2X_2$
Find values for $Var[Y_1]$, $Var[Y_2]$ and $COV[Y_1 Y_2]$ **(08 Marks)**

b. X is a random variable uniformly distributed between 0 and 3. Z is a random variable, independent of X, uniformly distributed between +1 and -1. U = X + Z, what is the pdf for U? **(08 Marks)**

c. Explain briefly the following random variables:
   (i) Chi-square Random variable.
   (ii) Raleigh Random variable. **(04 Marks)**

Module-3

5 a. With the help of an example, define Random process and discuss the terms Strict-Sense Stationary (SSS) and Wide Sense Stationary (WSS) associated with a random process. **(06 Marks)**

b. Two jointly wide sense stationary random process have the same functions of the form $x(t) = A\cos(w_0 t + \theta)$ and $y(t) = B\cos(w_0 t + \theta + \phi)$. Here A, B and ϕ are constants, θ is the random variable uniformly distributed between 0 to 2π. Find the cross correlation function $R_{XY}(t)$. **(06 Marks)**

c. Define the Autocorrelation function (ACF) of the random process X(t) and prove the following statements :
   (i) ACF is an even function.
   (ii) If X(t) is periodic with period T, then in the WSS case, ACF is also periodic with period T. **(08 Marks)**

OR

6 a. A random process is described by,
$X(t) = A\sin(w_c t + \theta)$
Where A and w_c are constants and where θ is a random variable uniformly distributed between $\pm\pi$. Is x(t) wide sense stationary. If not, then why not? If so, then what are the mean and the autocorrelation function for the random process? **(06 Marks)**

b. x(t) and y(t) are zero-mean, jointly wide sense stationary random processes. The random process z(t) is,
$z(t) = 3x(t) + y(t)$.
Find the correlation functions $R_Z(\tau)$, $R_{ZX}(\tau)$, $R_{XZ}(\tau)$ and $R_{YZ}(\tau)$. **(08 Marks)**

c. Assume that the data in the following table are obtained from a windowed sample function obtained from an ergodic random process. Estimate the autocorrelation function for $\tau = 0, 3$ and 6 ms, where $\Delta t = 3$ ms.

x(t)	1.0	2.2	1.5	−3.0	−0.5	1.7	−3.5	−1.5	1.6	−1.3
k	0	1	2	3	4	5	6	7	8	9

(06 Marks)

Module-4

7 a. Determine if the following set of vectors will be basis for R^3.
$u_1 = (1,-1,1), u_2 = (0,1,2), u_3 = (3,0,-1)$ **(05 Marks)**

b. Determine if the following sets of vectors are linearly independent or linearly dependent :
$v_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$. **(05 Marks)**

c. Apply Gram-Schmitt process to the vectors $v_1 = (2,2,1)$, $v_2 = (1,3,1)$ and $v_3 = (1,2,2)$ to obtain an orthonormal basis for $v_3(R)$ with standard inner product. **(10 Marks)**

OR

8 a. Determine Rank of the matrix A, $A = \begin{bmatrix} 1 & 3 & 9 \\ 4 & 1 & 3 \\ 9 & 4 & 12 \end{bmatrix}$. **(04 Marks)**

b. Solve Ax = b by least squares and find the projections of b on to the column space of A.
$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ **(06 Marks)**

c. Explain the following :
(i) Rank Nullity theorem.
(ii) Gram-Schmidt orthogonalization procedure. **(10 Marks)**

Module-5

9 a. Briefly explain the following :
(i) Cofactors of the determinant.
(ii) Symmetric matrix and its properties. **(04 Marks)**

b. If $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$. Find eigen values and corresponding eigen vectors for matrix A. **(08 Marks)**

c. Factor the matrix A into $P^{-1}AP$ using diagonalization and hence find D^4.
$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ **(08 Marks)**

OR

10 a. If $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ show that matrix A is positive definite matrix. **(04 Marks)**

b. Diagonalize the following matrix if possible :

$$A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$$

(06 Marks)

c. Factorize the matrix A into $A = U\sum V^T$ using SVD.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{bmatrix}.$$

(10 Marks)

* * * * *